**B. Transformations of Functions**

Basic functions such as *f*(*x*) = *x*2 can be thought of as building blocks for other functions.

For example, the graph of *g*(*x*) = (*x* – 2)2 + 1 has the same parabola shape as the graph of *f*, but its vertex is located at (2, 1). (Recall that quadratic functions were reviewed in [topic II-E](https://umuc.equella.ecollege.com/file/51ed41e5-be80-4110-8171-a40ed58c98af/1/MATH108-0609.zip/Modules/M1-Module_1/S3-Commentary.html#II.E.Quadratic).)

|  |  |
| --- | --- |
| To arrive at the parabola  *y* = (*x* – 2)2 + 1, shift the parabola *y* = *x*2 to the right by 2 units and upward by 1 unit.  The vertex has been shifted from (0, 0) to (2, 1).  Notice that  *g*(*x*) = (*x* – 2)2 + 1 = *f*(*x* – 2) + 1, so the graph of *g* involves a shift of the graph of *f* to the right by 2 units and upward by 1 unit. |  |

This example has illustrated both a horizontal and a vertical translation. Other types of transformations include reflection, vertical stretching and shrinking, and horizontal stretching and shrinking. The table below summarizes these types of transformations.

|  |  |
| --- | --- |
| **Transformations of Functions** |  |
| Vertical translation: *y* = *f*(*x*) ± *b*, for *b* > 0 | |
| The graph of *y* = *f*(*x*) + *b* is the graph of *y* = *f*(*x*) shifted upward *b* units. |  |
| The graph of *y* = *f*(*x*) –*b* is the graph of *y* = *f*(*x*) shifted downward *b* units. |  |
| Horizontal translation: *y* = *f*(*x* https://umuc.equella.ecollege.com/file/51ed41e5-be80-4110-8171-a40ed58c98af/1/MATH108-0609.zip/Modules/M1-Module_1/images/mod1hor.gif*b*),  for *b* > 0 | |
| The graph of *y* = *f*(*x*–*b*) is the graph of *y* = *f*(*x*) shifted rightward *b* units. |  |
| The graph of *y* = *f*(*x*+ *b*) is the graph of *y* = *f*(*x*) shifted leftward *b*units. |  |
| Reflection: *y* = –*f*(*x*) or *y* = *f*(–*x*) | |
| The graph of *y* = –*f*(*x*) is the reflection of the graph of *y* = *f*(*x*) across the *x*-axis. |  |
| The graph of *y* = *f*(–*x*) is the reflection of the graph of *y* = *f*(*x*) across the *y*-axis. |  |
| Vertical Stretching or Shrinking: *y* = *cf*(*x*) | |
| For |*c*| > 1, the graph of *y* = *cf*(*x*) is a vertical stretching of the graph of *y* = *f*(*x*). |  |
| For 0 < |*c*| < 1, the graph of *y* = *cf*(*x*) is a vertical shrinking of the graph of *y* = *f*(*x*). |  |
| Horizontal Stretching or Shrinking: *y* = *f*(*cx*) | |
| For |*c*| > 1,  the graph of *y* = *f*(*cx*) is a horizontal shrinking of the graph of *y* = *f*(*x*). |  |
| For 0 < |*c*| < 1, the graph of *y* = *f*(*cx*) is a horizontal stretching of the graph of *y* = *f*(*x*). |  |